NAME: Solutions.

There are 6 problems with total of 25 parts. Each PART is worth 2 points. You have 50 minutes. Good luck.

1. Suppose that a city sponsors a football bowl game every year. Over the years, city officials have come up with the following figures: The probability of having teams that will draw a large crowd is 0.75 and the probability of having good weather is 0.90. They also know that concession sales depend on various combinations of these factors, as shown in the table below.

a. Assuming that the team attractiveness and quality of weather are independent events, complete the column with probabilities:

good weather .9
bad weather .1
Attractive team .75
Unattractive team .25

Conditions	Consessions Revenue	Probability
Attractive team, good weather	200,000	$.9 \times .75 = .675$
Attractive team, bad weather	150,000	$.1 \times .75 = .075$
Unattractive team, good weather	100,000	$.9 \times .25 = .225$
Unattractive team, bad weather	50,000	$.1 \times .25 = .025$

b. Find the expected value for concession sales.

$200,000 \times .675 = 135,000$
$150,000 \times .075 = 11,250$
$100,000 \times .225 = 22,500$
$50,000 \times .025 = 1,250$

sum : 170,000

Expected value = \$170,000.

2. Combinations and Permutations.

a. Write down the formula for calculating ${}_nP_r$

$$ {}_nP_r = \underbrace{n(n-1)(n-2)\cdots}_{r\text{-many factors}} $$

b. Write down the formula for calculating ${}_nC_r$

$$ {}_nC_r = \frac{n(n-1)(n-2)\cdots}{r\cdot(r-1)(r-2)\cdots 2\cdot 1} $$

r-many factors on top.

c. How many permutations can you get from 5 different objects?

$5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$

d. In a toss of 8 coins, how many different ways could exactly 5 heads appear?

$$ {}_8C_5 = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 56. $$

e. What is the probability of getting exactly 5 heads in a toss of 8 fair coins?

$$ 56\cdot\frac{1}{2^8} = \frac{56}{256} = .219 = 21.9\% $$

f. In a toss of 8 coins, how many different ways could at least 5 heads appear?

$${}_8C_5 = 56$$

$${}_8C_6 = \frac{8\cdot 7\cdot \cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot 2\cdot 1} = \frac{8\cdot 7}{2\cdot 1} = 28.$$

$${}_8C_7 = 8$$

$$1+8+28+56 = 93$$

$${}_8C_8 = 1$$

g. What is the probability of getting at least 5 heads in a toss of 8 fair coins?

$$93 \cdot \frac{1}{2^8} = \frac{93}{256} = .363 = 36.3\%.$$

h. In a toss of 8 loaded coins with probability of heads = 0.8, what is the probability of getting exactly 5 heads?

$$56 \cdot 0.8^5 \cdot 0.2^3 = .1468 = 14.68\%.$$

i. In a toss of 8 loaded coins with probability of heads = 0.8, what is the probability of getting at least 5 heads?

$$56 \cdot 0.8^5 \cdot 0.2^3 + 28 \cdot 0.8^6 \cdot 0.2^2 + 8 \cdot 0.8^7 \cdot 0.2^1 +$$

$$+ 1 \cdot 0.8^8 = .1468 + .2936 + 0.3355 + 0.1678 =$$

$$= .9437 = 94.37\%.$$

3. Suppose that the equation for the line of best fit for some data is

$$y = 0.75x + 120, \qquad \text{with } R^2 = 0.81$$

a. What is the correlation coefficient?

$R = \sqrt{0.81} = 0.9.$

b. What does the correlation coefficient tell you about correlation between x and y?

x and y are positively correlated. I.e., if x increases we could expect for y to increase as well.

c. What is the predicted y value for an x value of 70?

$0.75 \cdot 70 + 120 = 172.5$

d. What x value might give a y value of 80?

$80 = .75x + 120$

$.75x = -40 \qquad x = \frac{-40}{.75} = -53.3.$

4. Refer to the Table 1 attached.

a. What is the likelihood of having a sample statistic of exactly 63% when the sample size is 1000?

$\frac{240}{1000} = 24\%.$

b. What is the likelihood of having a sample statistic within ±1% of the population parameter when the sample size is 2000?

$\frac{229 + 335 + 273}{1000} = \frac{837}{1000} = 83.7\%.$

5. Suppose 625 people were polled and 55% said that they would support funding for a new library.

a. Fill in the blanks in the following sentence: "We can be 95% sure that the percentage of population supporting funding for the new library is between __51__% and __59__%."

$$\frac{1}{\sqrt{625}} = \frac{1}{25} = 4\% \qquad 55-4=51 \quad 55+4=59$$

b. Are you confident that the funding for a new library will be approved? Why or why not?

The confidence interval (with 95% confidence level) is entirely above 50%, so we can be pretty confident that the funding for a new library would be approved.

Suppose, instead, that only 100 people were polled.

c. Fill in the blanks in the following sentence: "We can be 95% sure that the percentage of population supporting funding for the new library is between __45__% and __65__%."

$$\frac{1}{\sqrt{100}} = \frac{1}{10} = 10\% \qquad 55-10=45 \quad 55+10=65$$

d. Are you confident that the funding for a new library will be approved? Why or why not?

Since the confidence interval contains percentages below 50%, there is a good chance that the funding will not be approved.

e. How many people should be polled to limit the margin of error to ±1%?

$$\frac{1}{\sqrt{n}} = 1\% = \frac{1}{100}$$

$$\sqrt{n} = 100$$

$$n = 100^2 = \boxed{10\,000}$$

6. a. State the Fundamental Counting Principle.

If Act 1 can be performed in m ways and Act 2 can be performed in n ways, then a sequence Act 1 – Act 2 can be performed in $m \times n$ ways.

Explain the following concepts and support your explanations with examples.

b. Positive correlation.

Variables x and y are positively correlated if x and y change in the same direction (either both increase or both decrease).

Example: Age of a dragon and size of a dragon (They never stop growing).

c. Negative correlation.

Variables x and y are negatively correlated if x and y change in opposite directions (increase in one goes with decrease in the other)

Example: Latitude and temperature in January.