NAME: Solutions

There are 5 problems with total of 25 parts. Each PART is worth 2 points. You have 50 minutes. Good luck.

1. Consider an experiment of tossing one red die and one green die.

a. What is the probability of getting 3 on the red die?

$P(3 \text{ on red}) = 1/6$

b. What is the probability of getting 4 on the green die?

$P(4 \text{ on green}) = 1/6$

c. What is the probability of getting 3 on the red die and 4 on the green die?

Events are independent

$\Rightarrow \quad P(34) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}.$

d. Suppose for events A and B, we know values of $P(A)$, $P(B)$ and $P(A \text{ and } B)$. How would you calculate $P(A \text{ or } B)$?

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

e. What is the probability of getting 3 on the red die or 4 on the green die?

$P(3 \text{ on red or } 4 \text{ on green}) = P(3 \text{ on red}) +$

$+ P(4 \text{ on green}) - P(34) = \frac{1}{6} + \frac{1}{6} - \frac{1}{36} =$

$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}.$

2. Suppose we have a bag with 2 red balls and 5 blue balls. Consider an experiment of randomly drawing two balls from the bag WITHOUT replacement.

a. Give an example of an outcome of this experiment.

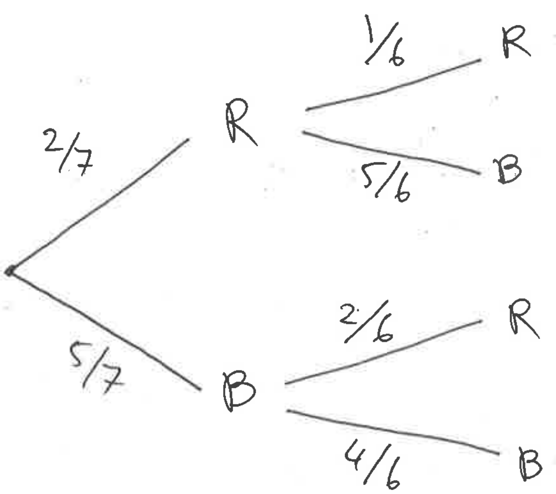
Red and Red

b. Give an example of an event that includes two or more outcomes.

Two balls have the same color. =

$= \{RR, BB\}.$

c. Draw a tree diagram illustrating this experiment. Make sure to include probabilities for each branch, list of all outcomes and corresponding probabilities.



Outcomes	Probabilities
RR	$1/6 \cdot 2/7 = 2/42$
RB	$2/7 \cdot 5/6 = \frac{10}{42}$
BR	$\frac{5}{7} \cdot \frac{2}{6} = \frac{10}{42}$
BB	$\frac{5}{7} \cdot \frac{4}{6} = \frac{20}{42}$

d. Find P(Balls are of different colors).

$P(\text{Balls are of different colors}) = P(RB) + P(BR) = \frac{10}{42} + \frac{10}{42} = \boxed{\frac{20}{42}}$

e. Find P(Balls are of the same color).

$P(\text{Balls are of the same color}) = 1 - P(\text{Balls are of dif. col.}) = 1 - \frac{20}{42} = \boxed{\frac{22}{42}}$

f. Find P(Both balls are red).

$P(RR) = \frac{2}{42}$

g. Suppose for independent events A and B, we know values of $P(A)$ and $P(B)$. How would you calculate $P(A \text{ and } B)$?

$P(A \text{ and } B) = P(A) \cdot P(B)$

h. Are the events "Balls are of the same color" and "Both balls are red" independent? (Answers from e, f and g might be useful).

$P(\text{Balls are of the same color } \underline{\text{and}} \text{ both are red}) =$

$= P(RR) = \frac{2}{42}$ $\qquad (e) \cdot (f) = \frac{22}{42} \cdot \frac{2}{42}$

Since $\frac{2}{42} \neq \frac{22}{42} \cdot \frac{2}{42}$, part (g) says that the events are <u>not</u> independent.

i. Find P(Red on the second draw).

$P(\text{Red on } 2^{nd}) = P(RR) + P(BR) = \frac{2}{42} + \frac{10}{42} = \boxed{\frac{12}{42}}$

j. Find P(Red on the second draw | Red on the first draw).

$P(\text{Red on } 2^{nd} \mid \text{Red on } 1^{st}) = \frac{1}{6}$ (read out from the tree diagram)

k. Considering answers from parts i and j, are the events "Red on the first draw" and "Red on the second draw" independent? Explain.

From part (i), probability of "Red on 2^{nd}" without knowing what happend on the first draw is $\frac{12}{42}$.

But part (j) tells us that if we know that Red ball has been drawn on the first draw probability of "Red on 2^{nd}" is $\frac{1}{6}$ which is not equal to $\frac{12}{42}$. So occurence of "Red on 1^{st}" changed probability of "Red on 2^{nd}" $\Longrightarrow$ events are <u>not</u> independent.

3. The following table shows gender and color of 10 puppies:

(a)

	Male	Female	Totals
Black	1	4	5
Brown	2	3	5
Totals	3	7	

(b)

a. What is the probability that a puppy is black given that it is female?

$$P(\text{Black} \mid \text{Female}) = \frac{4}{7}$$

b. What is the probability that a puppy is female given that it is black?

$$P(\text{Female} \mid \text{Black}) = \frac{4}{5}$$

c. What is the probability that a puppy is female or black?

$$P(\text{Female or Black}) = \frac{4+3+1}{10} = \frac{8}{10}.$$

4. Consider an experiment of tossing one nickel and one dime.

a. Simulate this experiment 10 times using the following random numbers:

8389 2438 7219 2120 0503 0958 0639 9506 0829 7445 2837 0985 7381 2883 4728

This means you must clearly state which numbers correspond to heads, which numbers correspond to tails, write out numbers corresponding to 10 simulations and translate the numbers back into heads and tails.

Heads – 0, 2, 4, 6, 8

Tails – 1, 3, 5, 7, 9.

1^{st} place – nickel

2^{nd} place – dime

10 simulations: 83, 89, 24, 38, 72, 19, 21, 20, 05, 03

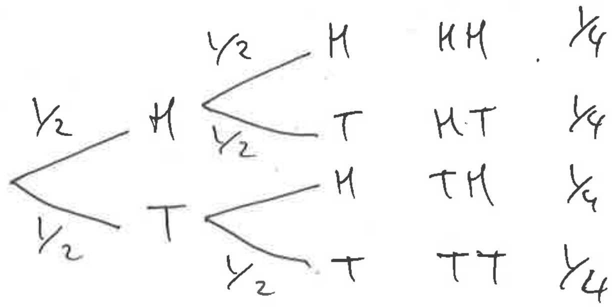
Translation: HT, HT, HH, TH, TH, TT, HT, HH, HT, HT.

b. What is the experimental probability of getting two tails?

We got only one outcome = TT out of 10 simulations.

$\Rightarrow$ Experimental $P(TT) = \frac{1}{10}$.

c. What is the theoretical probability of getting two tails?



Theoretical $P(TT) = \frac{1}{4}$.

5. Explain the following concepts and support your explanations with examples.

a. Disjoint events.

Two events are disjoint if it is not possible for them to occur at the same time

$\iff$ $P(A \text{ and } B) = 0$.

Example: Getting a 2 on one of the dice when tossing two dice and getting a sum of 11.

b. Biased sample.

Sample is biased if the method of obtaining the sample suggests that the sample might not reflect the population.

Example: Suppose you are trying to determine average height of students in your college. It is likely that your sample will not reflect the population if you only pick basketball players for your sample.

c. Stratified random sampling.

Method of sampling when the population is divided into groups and samples are picked randomly from each group.

Example: Again suppose you are trying to determine average height of students in your college. If you pick 100 female students and 100 male students at random, you will end up with a stratified random sample with two strata — males and females.