HW5 Solutions.

Section 32.1:

(2)

	$\le 10\%$	$\le 5\%$	$\le 2\%$
100	975	775	421
500	1000	987	771
1000	1000	1000	885
2000	1000	1000	974
3000	1000	1000	991

(3) a. $\frac{85}{1000} = 8.5\%$

b. $\frac{240}{1000} = 24\%$

c. $\frac{335}{1000} = 33.5\%$.

d. $\frac{421}{1000} = 42.1\%$.

(6) Using the $\frac{1}{\sqrt{n}}$ rule of thumb, there is a good chance (95% chance) that the population parameter lies in the interval $54\% \pm \frac{1}{\sqrt{400}}$ i.e. between 49% ~~and~~ 59%.

(7) a. $\frac{1}{\sqrt{1631}} \approx \frac{1}{40} = 2.5\%$. So there is a good chance that the population parameter is between 50.5% and 55.5%. Hence, it is very likely that the people will not vote for funding a new library.

b. But with 100 people, $\frac{1}{\sqrt{100}} = \frac{1}{10} = 10\%$, so there is a good chance that the number of people voting against the library measure will be between 43% & 63%. You should not feel as certain as in part a).

c. However if the sample statistic is 75%, then the population parameter should be between 65% and 85%, and so in this case 100 randomly selected people should be enough. Of course, the polsters do not just poll until the numbers are right. And using the $\frac{1}{\sqrt{n}}$ rule of thumb is risky, with a population parameter suggested by the 75%.

⑨ Getting 200 or more heads out of 300 coin tosses is more likely than 2 or more heads out of 3 coin tosses.

⑪ a. 400 (from $\frac{1}{\sqrt{n}} = 5\% = \frac{1}{20}$)

b. 10,000

c. 40,000.

Section 32.2:

① Yes, he could lose, because the confidence interval (49 to 59) contains 49%. He could ask the pollsters take a larger sample to get a smaller confidence interval, but of course then the poll may show him closer to 50%, so the confidence interval could still contain percents below 50. It is not possible to eliminate the uncertainty entirely, but he could get more accurate results by polling more people.

④ With the margin of error in mind, the percent of the population supporting a change would be in the 51% to 55% interval. If the matter were to be voted on, there is a good chance that there would be some changes to Social Security.

⑤ a. margin of error $= \frac{1}{\sqrt{9}} = \frac{1}{3} = 33\%$.

confidence interval: $(52-33, 52+33) =$ 19% to 85%.

b. margin of error: $\frac{1}{\sqrt{90}} = 10.5\%$

confidence interval: 41.5% to 62.5%

c. $\frac{1}{\sqrt{900}} = \frac{1}{\sqrt{30}} = 3.3\%$. 48.7% to 55.3%.

d. $\frac{1}{\sqrt{9000}} \approx 1\%$. 51% to 53%.

Section 33.1:

① $1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5$.

② Expected ~~number~~ of moves from 1st die + expected number of moves from the second die $= 3.5 + 3.5 = 7$.

③ P(bad weather) $= 1 - P(\text{good weather}) = 1 - 0.85 = 0.15$.
P(Unattractive team) $= 1 - 0.7 = 0.3$.

Since the events are independent we could calculate combined probabilities by multiplying:

conditions	Revenue	Probability	Rev × Prob
Attractive team, good weather	480 000	0.85 × 0.7	0.85 × 0.7 × 480000
Attr. team, bad weather	400 000	0.15 × 0.7	0.15 × 0.7 × 400 000
Unattractive team, good weather	320 000	0.85 × 0.3	0.85 × 0.3 × 320 000
Unattract. team, bad weather	200 000	0.15 × 0.3	0.15 × 0.3 × 200 000
		sum:	$\boxed{418,200}$

SLE 4:

a. $0.05 \times 0 + 0.15 \times 1 + 0.25 \times 2 + 0.30 \times 3 + 0.10 \times 4 + 0.15 \times 5 = 2.7$.

b. She can expect to get an average of 2.7 calls per day, over the long run.

Section 33.2:

① a. Important b. not important c. important
d. important e. not important.

② a. There are $5 \cdot 4 = 20$ permutations:

PQ	QP	QR	RQ	RS	SR
PR	RP	QS	SQ	RT	TR
PS	SP	QT	TQ	ST	TS
PT	TP				

b. There are $\frac{5 \cdot 4}{2 \cdot 1} = 10$ combinations:

PQ, PR, PS, PT, QR, QS, QT, RS, RT, ST.

(6) a. ${}_{10}C_3 \cdot \left(\frac{1}{2}\right)^{10} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} \cdot \frac{1}{2^{10}} = \frac{15}{128}.$

b. exactly 8 tails = exactly 2 heads : ${}_{10}C_2 \cdot \frac{1}{2^{10}} = \frac{10 \cdot 9}{2 \cdot 1} \cdot \frac{1}{2^{10}} =$

$= \frac{45}{1024}.$

c. ${}_{10}C_3 \cdot \frac{1}{2^{10}} + {}_{10}C_2 \cdot \frac{1}{2^{10}} + {}_{10}C_1 \cdot \frac{1}{2^{10}} + {}_{10}C_0 \cdot \frac{1}{2^{10}} = \frac{176}{1024} = \frac{11}{64}.$

d. In the long run, with many tosses of 10 honest coins, about 15 out of every 128 will have exactly 3 heads. (similar statements for b & c).

(7) $P(\text{all boys}) = P(\text{all girls}) = 1 \cdot \left(\frac{1}{2}\right)^4.$

$P(\text{exactly one boy}) = P(\text{exactly 1 girl}) = {}_4C_1 \cdot \left(\frac{1}{2}\right)^4 = 4 \cdot \left(\frac{1}{2}\right)^4 = \frac{1}{4} = 25\%.$

$P(\text{exactly 2 boys}) = {}_4C_2 \cdot \left(\frac{1}{2}\right)^4 = \frac{4 \cdot 3}{1 \cdot 2} \cdot \left(\frac{1}{2}\right)^4 = 6 \cdot \left(\frac{1}{2}\right)^4.$

In many future families with 4 children, about 25% will have exactly 1 boy. ....

You can make similar statements for other probabilities.

(8) $P(\text{all male}) = \left(\frac{2}{3}\right)^4$ $\quad P(\text{all female}) = \left(\frac{1}{3}\right)^4.$

$P(\text{exactly one male}) = {}_4C_1 \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^3 = 4 \cdot \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3$

$P(\text{exactly one female}) = {}_4C_1 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right) = 4 \cdot \left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)$

$P(\text{2 male \& 2 female}) = {}_4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2 = 6 \cdot \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2.$

(12) ${}_{10}C_8 \cdot 0.85^8 \cdot 0.15^2 +$

$+ {}_{10}C_9 \, 0.85^9 \cdot 0.15 +$

$+ {}_{10}C_{10} \cdot 0.85^{10} =$

$= 82\%.$

(10) a. ${}_{10}C_3 = 120$ $\quad$ b. ${}_6C_3 = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20.$ $\quad$ c. $\frac{20}{120}.$

d. ${}_4C_2 = 6$ $\quad$ e. ${}_{10}C_5 = 252.$

f. $\frac{20 \cdot 6}{252}$