Math 2870. Solutions for HW1.

27.1:

1. a. Not probabilistic, because the ~~gender~~ of the secretary of state is already decided

   Rephrase: The probability of the next secretary of state being a woman.

   b. Probabilistic.

   c. Not probabilistic, because the gender is already decided.

   Reformulation: see part d.

   d. Yes, probabilistic.

   e. Probabilistic.

   f. Not probabilistic, because Joe either did or did not eat pizza yesterday — it is already decided

   Reformulate: The probability that Joe will eat pizza tomorrow.

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4. a. Toss the penny, nickel, and dime together a large number of times and record outcomes.

   All outcomes: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. Here first place is for penny, second for nickel and third for dime

   b. Any of the above.

   c. Getting exactly two heads: HHT, HTH, THH

   The remaining 5 outcomes will not be in this event.

⑤ a. Toss three dice many times and record outcomes. We could could distinguish between the dice by coloring them or marking them. ②

Possible outcomes: { 111, 112, 113, 114, 115, 116,
121, 122, 123, ... }.

b. Say; 111.

c. An event might be tossing a sum of 13.
One outcome in this event would be 4 6 3.

d. 113, 131, 311, 221, 212, 122.

27.2:

④ Yes. $P(E)=0$ means E never happens.
For example E = Getting sum = 14 when tossing two dice

⑤ Yes. $P(E)=1$ means E always happens.
For example E = Getting sum ≤ 12 when tossing two dice.

⑥ No. Largest probability possible is 1 since an event cannot happen more often than always.

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9. a. Q, R, S, T, U, V. Each with probability $\frac{1}{6}$.

b. $W(\frac{1}{2})$, $X(\frac{1}{4})$, $Y(\frac{1}{8})$, $Z(\frac{1}{8})$

c. $E: \frac{1}{2}$, $F: \frac{1}{4}$, $G: \frac{1}{4}$.

d. Sample space (probabilities): $H(\frac{1}{6})$, $I(\frac{1}{3})$, $J(\frac{1}{2})$.

e. When spinning a spinner we use angle proportions to calculate probabilities, but while throwing darts what matters is area proportions. Therefore answers for a, b and d would not change but for c, $P(E) < \frac{1}{2}$ and $P(F) > \frac{1}{4}$.

13: a. The event not-E and event E would involve all of the outcomes, so $P(E) + P(\text{not-}E) = 1$

$\Rightarrow \quad P(\text{not-}E) = 1 - P(E)$.

b. $\frac{48}{52} = \frac{12}{13} = 1 - \frac{1}{13}$.

c. 1 − probability you got for thumbtack landing point up from activity 2 we did in class.

d. $\frac{3}{4}$

e. $\frac{5}{6}$

f. $\frac{4}{5}$, assuming all keys feel the same to touch.

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15 a. 5 equal sectors with angle $= \frac{360}{5} = 72°$.

b. Start with 6 equal sectors of 60° each and omit one of the dividing radii.

c. $x + x + x + 3x + 3x = 360 \Rightarrow 9x = 360 \Rightarrow x = 40°$.

sectors: 40, 40, 40, 120, 120.

d. Divide each spinner into 10 sectors (not necessarily equal) and label them with digits 0–9.

e. Each spinner could be marked into 5 sectors labeled 5–9.

(17) a. Experimentally, Draw a sample of, say, 20 and determine the fraction that is red.

b. Theoretically. Of the 6 outcomes each are equally likely and have probability $\frac{1}{6}$.

c. Experimentally, By choosing a large sample of students and determining the fraction that are from out of state.

d. Theoretically. There are 52 cards and 13 are hearts, so $P(\text{hearts}) = \frac{13}{52} = \frac{1}{4}$.

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(18). a. 900 b. 300

c. Yes, anything is possible while conducting an experiment. Some outcomes are just more likely than others.

d. Yes. Same as above.

e. 150 S's, 50 T's.

f. Perhaps something like: 3 out of 4 = how many out of 1200, or $\frac{3}{4}$ of the time you should get S, so $\frac{3}{4} \times 1200$ times.

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(19)

	a.	b.	c.	d.
P(Red) container 1	$\frac{2}{5}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
P(Red) container 2	$\frac{6}{13}$	$\frac{27}{72} = \frac{3}{8}$	$\frac{16}{43}$	$\frac{623}{2 \cdot 623} = \frac{1}{2}$
Conclusion	$\frac{6}{13} > \frac{2}{5}$ choose 2.	Same chance of winning. choose any	$\frac{3}{8} > \frac{16}{43}$ choose 1.	$\frac{1}{2} > \frac{3}{8}$ choose 2.

(22) P(landing in the park) = $\frac{\text{area of the park}}{\text{area of the circle}}$ =

$$= \frac{0.5 \times 0.25}{3^2 \cdot \pi} \approx 0.004 = .4\,\%.$$

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(24) a. P(Mammoths winning) = $\frac{3}{13}$.

b. P(**Dinosaurs** losing) = $\frac{6.7}{7.7}$

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(26) a. Odds of making a field goal are 2 to 1.

b. Odds of drawing a red ball from a bag are 7 to 5.

c. Odds of not getting a winning card are 7 to 23.

27.3:

(1) The experimental probabilities should be close to the theoretical probabilities of $\frac{1}{6}$ for each outcome. A greater number of repetitions should give a value closer to the $\frac{1}{6}$ value.

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(2) a. Assign 1 to Blue, 2 to Yellow, 3 and 4 to Green and 5 and 6 to Red.

Use random numbers table from the textbook and to write out $\underbrace{\text{60 digits}}_{\text{or 30 pairs}}$ from 1 to 6

translate your results into colors:

e.g. 25 would turn into Yellow, Red;

34 would turn into Green, Green. . .

b. Green and Red should show up the most because their probabilities are higher than those of Blue and Yellow.

c. $\dfrac{\#(\text{Green, Blue})\text{'s in your simulation}}{30}$

Theoretical probability $P(GB) = P(G) \cdot P(B) = \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}.$

d. $\dfrac{\#(\text{Green, Blue})\text{'s and } (\text{Blue, Green})\text{'s in your simulation}}{30}.$

Theoretically: $P(GB) + P(BG) - \underbrace{P(GB \text{ and } BG)}_{\text{not possible} \Rightarrow 0} =$

$= \frac{1}{18} + \frac{1}{18} = \frac{1}{9}.$

In this question order of Blue and Green does not matter so outcomes BG and GB are both counted.

e. $P(\text{not } GG) = 1 - P(GG) = \underbrace{1 - \frac{1}{3} \cdot \frac{1}{3} = \frac{8}{9}}_{\text{Theoretically}}$

or from your simulation

$P(\text{not } GG) = 1 - \dfrac{\#(GG)\text{'s in your simulation}}{30}.$

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(5) Simulations substitute the experiment because they are often easier and quicker but they still count for experiments. So probability obtained from simulations is experimental.