NAME: Solutions.

There are 14 problems with total of 50 parts. Each PART is worth 2 points. You have 2 hours and 30 minutes. Good luck.

1. In each part, write the property or properties being used.

a. $2x + 2y = 2(x + y)$.

Distributivity

b. $3a + (4 + 5a) = (3a + 5a) + 4$.

Commutativity and associativity of addition

2. Use the given property or properties to rewrite each expression.

a. $y + (-y + 12)$; associativity, existence of additive inverses, identity of ~~multiplication~~ addition.

$(y + (-y)) + 12 = 0 + 12 = 12$

b. $x4$; commutativity of multiplication.

$4x$

3. Patterns.

a. Write the next four terms of the sequence and the 100th term of the sequence:
5, 8, 11, 14, 17.

+3 +3 +3 +3

$\boxed{20, 23, 26, 29}$

100 th: $5+3\times 99 = 5+297 = \boxed{302}$

b. Write the next four terms of the sequence and the 50th term of the sequence:
$\underbrace{2, A, 3,}_{\text{gets repeated.}}$ 2, A, 3.

$\boxed{2\ A\ 3\ 2}$

50^{th}: $50 = 48+2 = 3\times 16 + 2$ = 3 full building blocks + 2 extra.

$\boxed{50^{th}: A}$

c. What is the first number in the arithmetic sequence that has a common difference of 4 and has 96 as its 20th term.

$d=4$: $a + 4\times 19 = 96$

$a = 96 - 4\times 19 = 96 - 76 = 20.$

$\boxed{\text{The first term is } 20}$

d. What digit is in ones' place in the calculated form of 3^{280}?

$\underset{3}{\overset{\shortparallel}{3}}, \underset{3^2}{\overset{\shortparallel}{9}}, \underset{3^3}{\overset{\shortparallel}{27}}, \underset{3^4}{\overset{\shortparallel}{81}}, \underset{3^5}{\overset{\shortparallel}{243}}, \underset{3^6}{\overset{\shortparallel}{729}}, \underset{3^7}{\overset{\shortparallel}{2187}}, \underset{3^8}{\overset{\shortparallel}{6561}}$

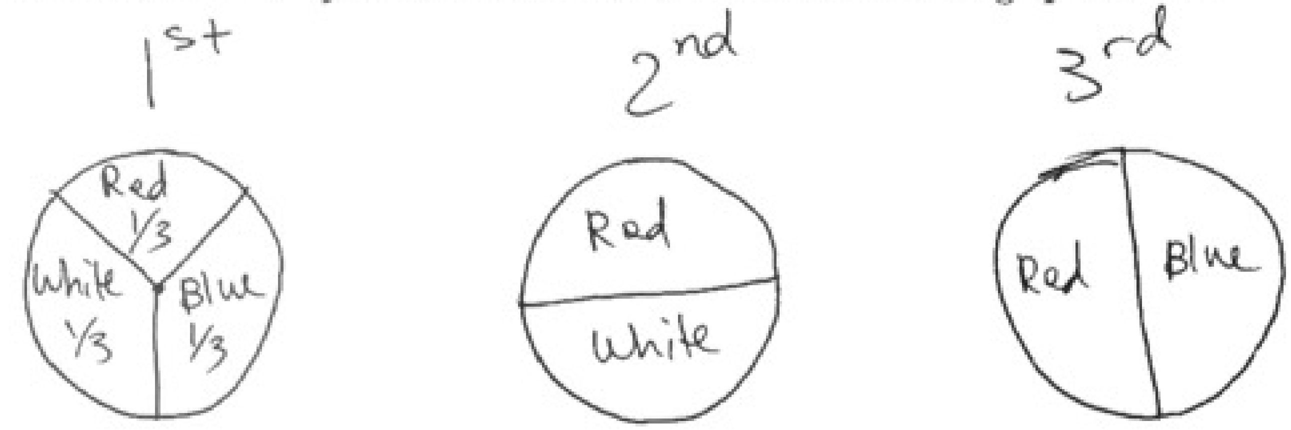
Ones' digits: 3, 9, 7, 1, 3, 9, 7, 1...

So 3971 gets repeated over and over.

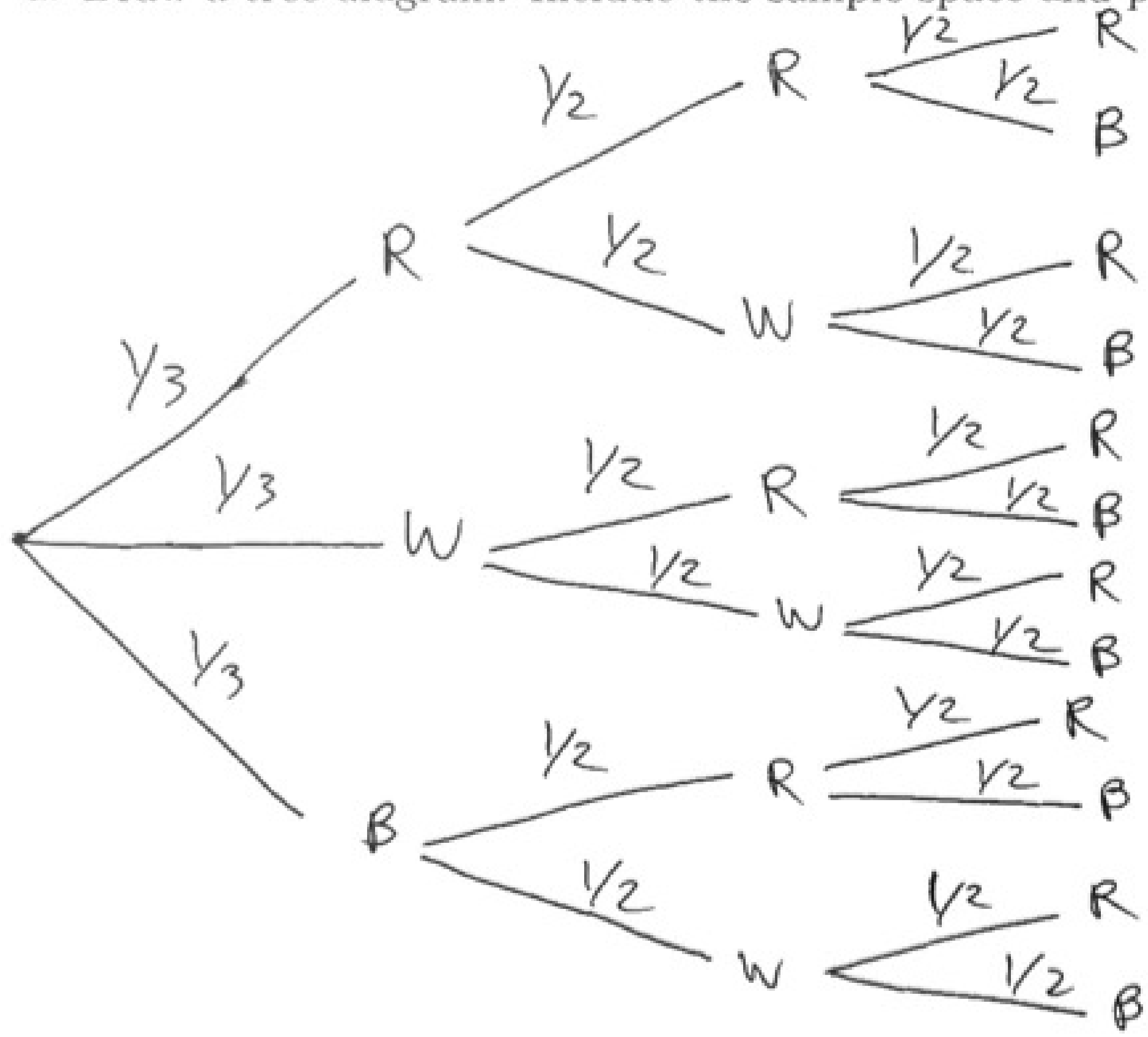
$280 = 4\times 70$ = 70 full building blocks.

$\boxed{\text{So 1 will be in ones' place in } 3^{280}.}$

4. Given the three spinners below answer the following questions.



a. Draw a tree diagram. Include the sample space and probabilities.



RRR	$\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{12}$
RRB	$1/12$
RWR	$1/12$
RWB	$1/12$
WRR	$1/12$
WRB	$1/12$
WWR	$1/12$
WWB	$1/12$
BRR	$1/12$
BRB	$1/12$
BWR	$1/12$
BWB.	$1/12$.

b. What is the probability of getting all the same color?

The only outcome with all 3 having the same color is RRR

so $P(\text{all same color}) = \frac{1}{12}$.

c. What is the probability of one or more red?

$P(\text{no red}) = P(WWB, BWB) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$.

$P(\text{1 or more red}) = 1 - \frac{1}{6} = \frac{5}{6}$

d. What is the probability of reds on both Spinner 1 and Spinner 2?

$$P(\text{Red on } 1^{st} \text{ and } 2^{nd}) = P(RRR, RRB) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}.$$

e. What is the probability of red on all three spinners given red on both Spinner 1 and Spinner 2?

$$P(RRR \mid \text{Red on } 1^{st} \,\&\, 2^{nd}) = \frac{P(RRR)}{P(RRR, RRB)} = \frac{1/12}{1/6} = \frac{1}{2}$$

f. What is the probability of RWR and all the same color? cannot happen at the same time

$$P(RWR \text{ and all same color}) = 0.$$

g. What is the probability of RWR or all the same color?

$$P(RWR \text{ or all same color}) = P(RWR, RRR) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}.$$

h. What is the probability of RWR and at least 2 reds?

$$P(RWR \text{ and at least 2 reds}) = P(RWR) = \frac{1}{12}$$

i. What is the probability of RWR or at least 2 reds?

$$P(RWR \text{ or at least 2 reds}) = P(RRR, RRB, RWR, WRR, BRR) =$$

$$= 5 \times \frac{1}{12} = \frac{5}{12}$$

5. Which of the following are disjoint events:

a. On a flip of one coin: getting heads and getting tails.

Disjoint

b. On a flip of two coins: getting at least one heads and getting the same on both coins.

Not disjoint (outcome HH satisfies both conditions).

1. Which of the following are independent events:

a. Getting a college degree and increase in salary.

Not independent

b. First child being a boy and the second child being a girl.

Independent.

6. In an experiment you are to draw one ball from a container without looking. You win if you draw a red ball. For each part, which container provides better chances of winning?

a. Container 1: 2 reds 3 blues;
Container 2: 3 reds 5 blues.

C1: $P(\text{red}) = \frac{2}{5} = 0.4$

C2: $P(\text{red}) = \frac{3}{8} = 0.375$

$0.4 > 0.375$

so C1

b. Container 1: 2 reds 3 blues;
Container 2: 10 reds 15 blues.

C1: $P(\text{red}) = \frac{2}{5} = 0.4$

C2: $P(\text{red}) = \frac{10}{25} = \frac{2}{3} = 0.4$

$\boxed{\text{Chances are the same}}$

7. For each part separately, generate a data set with at least 5 values with a given condition.

a. The box plot has only one box.

Want: Median = 1st quartile

$1, \underbrace{2 \quad 2,} 3 \quad 5$

b. The box plot has no right whisker.

Want: 3rd quartile = high

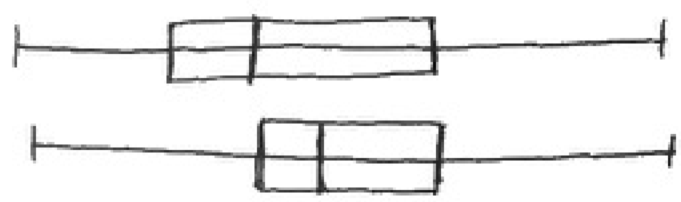
$1 \quad 2 \quad 3 \quad \underbrace{4 \quad 4}$

c. The box plot has no whiskers.

Want: Low = 1st quartile and 3rd quartile = high

$\underbrace{1 \quad 1,} \ 2 \ \underbrace{3 \quad 3,}$

8. Consider the following two box plots.



a. Give two characteristics that differ for the two data sets represented by the box plots.

The median is higher for the second data set.

The midde 50% is more spread out in the first data set.

b. Give two characteristics that are the same for the two data sets represented by the box plots.

The ranges are about the same.

The 3rd quartiles are very close.

9. Suppose Lisa is sitting at a booth at a carnival to offer you the following game: if you roll a pair with two dice you win $2. The game costs $1 to play.

a. What is the expected value of this game?

$P(\text{pair}) = P(11, 22, 33, 44, 55, 66) = \frac{6}{36} = \frac{1}{6}$

Expected value: $\frac{1}{6} \times \underbrace{(\$2 - \$1)}_{\text{net winnings}} + \frac{5}{6} \times \underbrace{(\$0 - \$1)}_{\text{net winnings}} = \frac{1}{6} - \frac{5}{6} = \frac{4}{6} = -67¢.$

Over the long run the player may expect to lose 67¢ per game

b. How much money can Lisa expect to make from 100 games?

$100 \times 67¢ = \$67$

10. In each case decide which sampling method best describes the given situation (circle one).

a. Stand by the science building and interview first 50 students that come out.

Convenience cluster simple random

b. Write names of each 30 of the female students in the class on pieces of paper and put them in one hat, put names of 20 male students in the class on pieces of paper and put them in another hat. Then pull out 3 names from the first hat and 2 names from the second.

Systematic simple random stratified random

c. Interview tenants of every even-numbered building.

Systematic cluster simple random

11. a. For events A and B write down the formula relating following probabilities: $P(B)$, $P(A \text{ and } B)$ and $P(A|B)$.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

b. Calculate the probability $P(A|B)$ when:
$A =$ getting a pair in a roll of two dice.
$B =$ getting sum of 4 in a roll of two dice.

$$P(B) = P(13, 31, 22) = \frac{3}{36} = \frac{1}{12}.$$

$$P(A \text{ and } B) = P(22) = \frac{1}{36}.$$

$$P(A|B) = \frac{1/36}{1/12} = \frac{12}{36} = \boxed{\frac{1}{3}.}$$

c. What is the relation between $P(A)$ and $P(A|B)$ when A and B are independent?

$$P(A) = P(A|B).$$

d. Using part c. decide whether or not the following events are independent.
$A =$ getting sum of 4 in a roll of two dice.
$B =$ getting a pair in a roll of two dice.

$$P(A) = P(13, 31, 22) = \frac{3}{36} = \frac{1}{12}.$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(22)}{P(11, 22, 33, 44, 55, 66)} = \frac{1/36}{6/36} = \frac{1}{6}.$$

Since $\frac{1}{12} \neq \frac{1}{6}$, two events are $\boxed{\text{not independent}}$

12. Refer to table 1. a. What is the likelihood of having a sample statistic within ±2% of the population parameter when the sample size is 1000?

$$116 + 226 + 240 + 204 + 99 = 885$$

$$\text{likelyhood} = \frac{885}{1000} = 88.5\%.$$

b. What confidence interval and what confidence level can you obtain from part a?

Confidence interval : 63−2 to 63+2 , or 61% to 65%.

Confidence level : 88.5%.

13. Permutations and combinations.

a. Suppose a bag contains 8 balls. How many different choices of 4 balls are possible?

$${}_8C_4 = \frac{\cancel{8}\cdot 7\cdot \overset{2}{\cancel{6}}\cdot 5}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = \frac{7\cdot 2\cdot 5}{1} = 70.$$

b. Suppose a bag contains 5 balls. How many different choices of 4 balls are possible?

$${}_5C_4 = \frac{5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 5.$$

c. If a bag of 8 balls contains 5 blue balls and 3 yellow balls, what is the probability that a draw of 4 balls will give all blue balls?

$$\frac{5}{70} = \frac{1}{14}.$$

d. If a bag of 8 balls contains 5 blue balls and 3 yellow balls, what is the probability that a draw of 5 balls will give 4 blue balls and 1 yellow ball?

$${}_3C_1 = \frac{3}{1} = 3. \qquad {}_8C_5 = \frac{8\cdot 7\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 56.$$

$$P(4\text{ blue \& 1 yellow}) = \frac{5\times 3}{56} = \boxed{\frac{15}{56}}$$

14. Concepts: explain and give an example.

a. Sample space.

For a given experiment, the sample space is the collection of all possible outcomes.

Example: Toss of a coin gives a sample space $\{H, T\}$.

b. Event.

For a given experiment, an event is any subset of the sample space.

Example: In a toss of two coins, getting two of the same would be an event $\{HH, TT\}$.

c. Sample statistic.

Sample statistic is a figure obtained by examining a sample of the population, rather than the whole population.

Example: Suppose you wish to measure average height of your classmates. If you randomly pick 5 students out of total of 30 in your class and measure their heights, then resulting average would be a sample statistic.

d. Population parameter.

Population parameter is a figure obtained by examining the whole population.

Example: If, in the example from part c, you instead measure heights of all 30 students, then the resulting average would be the population parameter.

e. Conditional probability.

$P(A|B)$ = probability of A given the condition that B has occured.

Example: Probability of getting a pair in a roll of two dice given the sum is 4 is equal to $1/3$, as calculated in problem 11 part b.

f. Experimental probability.

Experimental probability is a probability obtained by conducting the experiment many times by calculating the frequency of an event occuring out of all tries.

Example: If we roll a die 100 times and get 3 20 times, Then experimental probability of rolling a 3 $= \frac{20}{100} = \frac{1}{5}$.

g. $\frac{1}{\sqrt{n}}$ rule of thumb.

If the population parameter is expected to be close to 50%, $\frac{1}{\sqrt{n}}$ gives the margin of error for a sample statistic, where n is the sample size.

For example, if we interview 10,000 people and 58% says they like green tea, then you can be pretty sure that actual population parameter is within $\frac{1}{\sqrt{10,000}} = \frac{1}{100} = 1\%$ of 58% (i.e. between 57% and 59%).